

ABSTRACT – Mathematics Visualization Project

Current mathematical texts place excessively high cognitive overhead on their readers, which can lead to increased frustration, reduced understanding, reduced memory, and general fear and dislike of math. In particular:

- Readers must know and remember the meaning of every symbol represented, and often those symbols are only defined once in a different part of the text. Even worse, different symbols may be used to describe the same variable, or different variables may share the same symbol.
- Readers must remember the implications of previously presented assumptions, definitions, and derivation steps, all of which are not visible and may not be easily findable. Furthermore assumptions may be changed as the text progresses.
- Readers must often infer mathematical derivation steps that the authors felt were too detailed or too obvious to devote space to, which can be challenging for all levels of math capability, but particularly frustrating to those who are not comfortable with math.
- Readers must face usage of equations or terms that are not defined but rather referenced in external sources that are not easily accessible, but are critical to the understanding of the current text.

Many of these challenges are due to inherent limitations of the printed communication medium. However, common electronic versions of mathematical texts have not practically solved the problems we describe. Either they are perfect copies of the printed mathematical texts (i.e., PDF), or they are completely new paradigms for printing and publishing mathematical texts that require dramatic changes in the way that current publishing works to adopt the new electronic formats. Such approaches require too much cost to the stakeholders of the publishing process. Yet even if such approaches were practical, even the most radically different electronic mathematical communication media that have been proposed do not sufficiently address all the cognitive overhead issues that we have described. In fact, many of the proposed new electronic formats for texts make heavy use of hypertext. We argue that hypertext as a user interface leads to significant loss of context and disorientation on the part of the reader that is trying to understand mathematical equations.

We are in the beginning stages of developing technology that significantly reduces the cognitive overhead faced by readers of all levels when trying to understand math. This technology provides innovative solutions that will enable the creation of mathematical texts that will:

- *reduce the amount of working memory required* by a user to understand mathematical derivations.
- *reduce the amount of visual context switching required* by a user to understand mathematical derivations
- *adaptively provide different levels of mathematical derivation detail* that are appropriate to the readers current learning needs.
- *automatically generate visual animations of derivation steps* to provide visual accompaniments to text explanations.
- *integrate into current publishing workflows with little to no additional work by authors or publishers.*

This paper surveys the state of the art in digital mathematics textbooks and learning environments and describes our plans for a system that will not only solve the problems we have described, but will do so within the current textbook publishing paradigm.

There are many steps that could be taken to seek improvements in mathematical comprehension among our citizens, but most of them require continued increased ongoing investment, such as increased testing, teacher salaries, tutoring, or teaching training. We would like to identify areas for up-front investment that would lead to continued long-term payoff in the form of increased mathematical comprehension of our citizens.

We are proposing software technology that could dramatically increase our ability to communicate mathematical concepts, yet would require minimal changes to the current processes for communicating mathematics, and minimal to no additional work for the current stakeholders.

Our approach focuses on creating technology that automatically transforms existing mathematical texts into a digital form that presents mathematical equations and derivations in a way that reduces the cognitive overheads imposed on learners by the limitations of printed text.